XVIII. *Experiments on the Directive Power of large Steel Magnets, of Bars of magnetized Soft Iron, and of Galvanic Coils, in their Action on external small Magnets. By* George Biddell Airy, *Astronomer Royal, C.B., P.R.S.—With Appendix, containing an Investigation of the Attraction of a Galvanic Coil on a small Magnetic Mass. By* James Stuart, *Esq., M.A., Fellow of Trinity College, Cambridge.*

Received January 6,—Read February 8, 1872.

The only experiments with which I am acquainted tending to throw light upon the distribution of magnetic power in the different parts of a steel magnet are some very imperfect ones by Coulomb in the French Memoirs for 1789 and other years. It appeared to me that it might be desirable to make experiments of a rather more extensive character, and to add some measures of the magnetic effect of galvanic currents, both directly by their immediate action, and indirectly by the amount of magnetic power which they produce inductively in soft iron.

For the measure of permanent magnetism I selected a bar magnet 14 inches long, 1·4 inch broad, 0·35 inch thick; it has not been touched by a magnet for several years, and is likely to be in a state of very permanent magnetism. For the galvanic currents a cylindrical coil was used 13·4 inches long, 1·4 inch in external diameter, and about 0·9 inch in internal diameter; it has, I believe, four layers of wire, each layer having 160 revolutions of the wire. The battery used with it consisted of three cells, with sulphuric acid diluted in the proportion of 1 to 6; the plates were of zinc and graphite, each exposing on each side about 8 square inches; the circuit was always completed about half an hour before the experiments were begun, and a delicate galvanometer was placed in circuit by which the steadiness of the current was established. A core of iron 0·8 inch in diameter and of the same length as the coil, removable at pleasure, fits well in the inside of the coil; the iron is quite soft, and can with ease be entirely freed from any subpermanent magnetism.

The first step in the experiment was to neutralize terrestrial magnetism within the area of magnetic experiment. For this purpose two powerful 2-feet magnets were placed below the table on which the experiments were made, with their red or north-seeking ends directed to the magnetic north, at a distance (determined by trial) such that the experimental compass was sensibly uninfluenced by terrestrial magnetism. It is possible that some small residual magnetism was perceptible in the comparison with the feeble galvanic action, but none could be certainly discovered in the other experiments.

The compass used for register of the magnetic action is a small and very lively pocket-compass, with needle 1·0 inch long, not loaded with a card. The box of this compass is circular, and when positions had been selected for the centre of the compass (as will be mentioned), a circle somewhat larger than the compass-box was described in pencil with each of those positions for centre; and the compass could then be planted with its centre very accurately placed above the intended point.

The compass-positions were thus prepared:—Upon a sheet of strong paper the plan of the magnet, 14 inches by 1·4 inch, was laid down. On each side were drawn two parallel lines of the same length as the magnet, at distances respectively 1·5 inch and 3·0 inches from the near edge of the magnet; these lines were divided each into ten equal parts, and thus in each line eleven points were obtained at intervals of 1·4 inch. From each of the four angles of the magnet as centre, two quadrants were swept—one with radius at 1·5 inch, at whose extremity and bisection points were taken for the compass-centre; and one with radius 3·0 inches, which was twice bisected, and of which the extreme point and the three bisection-points were taken for the compass-centre. These points were used for the magnet both with its edge and with its flat side towards the compass. A similar process was adopted in using the galvanic coil, with this difference only, that the longitudinal separation of the points taken for compass-centre was only 1·34 inch.

A solid piece of wood was provided, in which was cut a concave channel, less than half a cylinder, such that when the galvanic coil, or the large magnet with its flat side towards the compass, was laid in the channel, its axis was sensibly at the same height as the needle of the small compass. With the magnet's edge towards the compass, that condition was sufficiently secured by merely laying its flat side upon the board. The paper with station-points, being laid in proper position upon the board and secured by nails, was cut along the middle of the channel and crosswise at its ends, so that it could be bent down into the channel to permit the magnet or coil to take its proper position; when observations were finished, the paper was detached from the board, and the edges which had been cut were re-united by cementing a piece of paper behind.

The observation (as will be seen) consisted, in every case, of observation of the direction taken by the small needle. And this observation was made solely by the eye. The observer, looking endways of the small needle, made two pencil dots upon the paper, corresponding to the line of the needle-axis produced as it appeared to his eye. If, from erroneous position of the eye, a parallactic error is produced in the position of the two pencil dots, this error is detected as soon as the compass has been removed and an attempt has been made to draw a line of direction through the station-point of the compass; and, to correct it, all that is required is, to draw through the station-point a line parallel to the line joining the two dots. The whole of this operation is extremely accurate.

For measuring the intensity of the magnetic force exerted on the compass-needle, I determined, after consideration, to adopt the statical method; that is, to place a constant magnet in a definite position above the compass-needle, with its magnetic axis transversal to the direction which the compass-needle had taken before the constant magnet was introduced, and to observe the deflection produced. The measure of the force of the large magnet was then the cotangent of the angle of deviation. The observation of the deflected needle by dots &c. was the same as before; but the angle of deviation was never measured by degrees. Instead of that measure, a circle upon semitransparent paper was graduated by cotangents, and thus the measure of the force of the large magnet was read off at once.

The arrangements in this state were confided to Mr. CARPENTER, Assistant of the Royal Observatory, by whom all the subsequent arrangements were planned and all the observations were made. I need not say that they were made with the utmost skill and delicacy.

A small frame was constructed, carrying a floor at a definite position about 1·8 inch above the compass-needle. As it was my object to make the observations at small distances from the great magnet, where its power is great, it was necessary to use a large power in the deflecting magnet. Mr. CARPENTER selected a horse-shoe magnet about 4 inches long, consisting of sixteen plates, each 0·06 inch thick; these were retouched a few days before they were used. From the consistency of the results obtained at the beginning and end of each circuit of the great magnet, I am entitled to conclude that no sensible change took place in the magnetism of the horse-shoe magnet. The magnet was placed in a vertical position, its two poles resting on the raised floor above mentioned. In all cases the deflecting magnet was used in the two positions to produce deflection right and deflection left.

These arrangements sufficed for observation of the powers of the great magnet in both positions, and also for observation of the galvanic coil carrying the soft iron core, the intensity of the battery having been in some measure adjusted to make the power of the coil with core not very different from that of the magnet. But when the coil was used without core, the force was so enormously reduced that the arrangement which applied well in the other cases failed totally in this. A small magnet was then used, 1·25 inch long, not very strongly magnetized; its deflecting power was compared with that of the horse-shoe magnet in the following way:—The small compass being under the influence of the earth's magnetism, the horse-shoe magnet and the small magnet were successively placed on the raised floor above mentioned, then 0·5 inch higher, then 1·0 inch higher, and the cotangents of deflection were compared. Thus the following proportions were obtained:—

Magnets upon the raised floor . . . . $\frac{\text{power of small magnet}}{\text{power of horse-shoe magnet}}=\frac{1}{107}$.

Magnets 0·5 inch above the raised floor . $\frac{\text{power of small magnet}}{\text{power of horse-shoe magnet}}=\frac{1}{136}$.

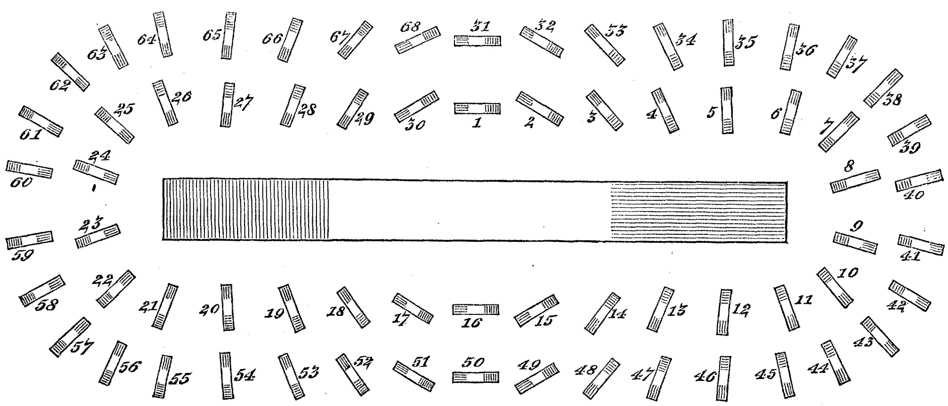
Magnets 1·0 inch above the raised floor . $\frac{\text{power of small magnet}}{\text{power of horse-shoe magnet}}=\frac{1}{125}$.

With so great inequality the results are necessarily irregular. I use $\frac{1}{120}$ as the proportion for comparison, without asserting that it is accurate. All results obtained for the coil without core ought therefore to be divided by 120, to make them comparable with the other results.

The results obtained for the direction of magnetical force now consisted of lines drawn upon paper. Upon examining these, some very small irregularities were found—generally of systematic character, partly arising from minute failure in the neutralization of terrestrial magnetism, partly from a difference in the intensity of the poles of the great magnet; these were eliminated by the following graphical process:—The paper was bent upon its longitudinal axis, and exposed to a strong light passing through the two folds of paper; the lines drawn upon both sides of the magnet or coil were visible, and a mean line bisecting the small angle between each pair was easily drawn. Then the paper was unfolded and was bent upon its transversal axis, and a similar operation was performed upon the mean lines mentioned above. Thus for one fourth part of the circumference of the magnet a series of lines was obtained representing the mean of the four parts; these mean lines, repeated for the four divisions of the magnet's or coil's circumference, are alone used in further graphical deductions and in the subjoined figure.

The results, however, for magnitude of force were obtained in numbers. The means of these were taken in an analogous order—first taking the sums of those on opposite sides of the magnet or coil, then taking the sums of the last-found sums for opposite ends. The division by 4 was omitted; and thus the numbers in the Table below give the value of 400 × cotangent of deviation. At two stations the proximity of the coil-terminals made it difficult to obtain actual observations; but there was no difficulty in supplying them conjecturally, with great confidence in their accuracy.

The diagram below was drawn carefully to represent the positions taken by the small magnet when the edge of the large magnet is presented to the small magnet. The same diagram will serve, almost without perceptible error, for the case when the flat side of the large magnet is presented to the small magnet, or for the case of a galvanic coil



inclosing an iron core; but it will not apply to the case of a galvanic coil without an iron core: for that case the axis of the small magnet in the positions numbered 35, 5, 12, 46, and all to the right of it must be directed almost exactly to the centre of the right-hand end of the magnet, and a similar direction must be made at 65, 27, 20, 54 in respect of the left end, with corresponding changes for intermediate stations.

The angles of position were never measured, but they are fully taken into account in the subsequent resolution of forces into longitudinal and transversal components.

The following Table exhibits the total force at each station, as ascertained by the operations described above. It will be remembered that the numbers for the "Galvanic coil with iron core" are not necessarily on the same scale as those for the "Large magnet," and that the numbers for the "Galvanic coil without core" must be divided by 120 to make them comparable with those for the "Galvanic coil with iron core."

Total force acting on the small magnet at each station.

No. of Station.				Large magnet presenting its edge.	Large magnet presenting its flat side.	Galvanic coil with iron core.	Galvanic coil without core.
1	16			274	250	310	216
2	15	17	30	326	293	330	240
3	14	18	29	408	363	413	333
4	13	19	28	566	480	515	550
5	12	20	27	678	542	634	1000
6	11	21	26	622	480	585	1470
7	10	22	25	513	454	565	1925
8	9	23	24	600	584	840	2700
31	50			160	160	164	184
32	49	51	68	163	165	180	192
33	48	52	67	191	183	195	225
34	47	53	66	224	200	221	305
35	46	54	65	235	217	227	400
36	45	55	64	211	197	215	427
37	44	56	63	193	180	200	444
38	43	57	62	175	173	195	485
39	42	58	61	181	177	208	583
40	41	59	60	201	193	227	690

Perhaps the following points are worthy of present notice:—

1. Remarking that, in the experiment in which the edge of the large magnet is presented to the small magnet, the distance of the small magnet is in each circuit the same at every station, it will be seen that the greatest directive force is not longitudinal at the end of the magnet, but transversal, at about $\frac{1}{10}$ part (or probably less) of the length from the end of the magnet. There is, however, a diminution and then an increase in proceeding from either of these positions to the other, and the directive force opposite the middle of the magnet's length is less than either of them; so that, in making the entire circuit of the magnet, there are six maxima and six minima.

2. When the flat side is presented to the small magnets, the same statement holds for the outer circuit, but not for the inner circuit.

3. With increase of distance, the diminution of force is much more rapid at the end than at the side of the large magnet.

4. The law of effect of a soft iron bar surrounded by a galvanic coil differs, but not greatly, from that of the large magnet presenting its edge. It would seem not improbable that this may depend partly on the effect of the coil which incloses the iron bar; and, if so, the law for a soft iron bar approaches still more to that of a magnet.

5. The exhibition of the effect of the magnetic coil alone is worthy of careful examination. The first thing which will strike the eye is the astounding increase of power produced by the insertion of the soft-iron core. At the sides of the magnet, where the measures of force for the coil alone are 1·5 and 1·8, those for the coil with core inclosed are 164·0 and 310·0; at the ends, where the coil alone gives 5·75 and 22·5, the coil with core included gives 227·0 and 840·0.

6. The law of magnitude of forces for the coil alone differs greatly from that of a steel magnet. In the inner circuit the proportion of the force at the end to force at the middle of length is, for the steel magnet $\frac{600}{274}$, for the coil $\frac{2700}{216}$; in the outer circuit they are $\frac{201}{160}$ and $\frac{690}{184}$.

7. Still more remarkable is the difference in the law of direction of the forces near the ends. Using the term "pole" to denote that point near the extremity to which the directions of forces rudely converge, the pole of the steel magnet is within the magnet, and distant from the end by about $\frac{1}{12}$ of the magnet's length: but the pole of the galvanic coil is absolutely at its end; indeed some of the experimental directions of force fall a little beyond the end.

It is evident, from the remarks of Nos. 6 and 7, that a magnet cannot in any wise be represented as a system of revolving galvanic currents, with an equal number of circuits at every part of its length.

With the view of presenting the results in the form which may probably be found most advantageous for comparison with the conclusions from any future theory, I have resolved the forces into rectangular directions, parallel and transversal to the axis of each magnet, by the following graphical process. Upon each mean line of direction of force (ascertained as is described above) I have laid down the mean measure of the force (as found above), and upon this measure as hypotenuse I have constructed a right-angled triangle, the lengths of whose sides give the two forces. From the nature of the preceding operations, it is only necessary to form these numbers for one quadrant of each magnet. The results are given in the following Tables. The centre of the large magnet or coil is in every case the origin of coordinates of the external magnetic point on which the action of the large magnet &c. is estimated—the axis of the longitudinal ordinate being the axis of the magnet, and the axis of the transversal ordinate being normal to it. The powers are estimated as those of the red end of the large magnet operating on a small external mass of red magnetism. It will be remembered that, for the galvanic coil without core, all the numbers must be divided by 120.

Large Bar Magnet.

For attracted point.		Edge towards small magnets.		Flat side towards small magnets.	
Longitudinal ordinate.	Transversal ordinate.	Longitudinal force.	Transversal force.	Longitudinal force.	Transversal force.
0·0	2·2	−274	0	−250	0
1·4	2·2	−283	+161	−260	+137
2·8	2·2	−262	+315	−236	+276
4·2	2·2	−198	+530	−182	+444
5·6	2·2	− 56	+678	− 36	+540
7·0	2·2	+216	+585	+158	+451
8·08	1·76	+367	+360	+325	+315
8·5	0·7	+580	+166	+552	+184
0·0	3·7	−160	0	−160	0
1·4	3·7	−147	+ 73	−149	+ 72
2·8	3·7	−127	+142	−123	+136
4·2	3·7	− 88	+205	− 79	+185
5·6	3·7	− 19	+235	− 12	+217
7·0	3·7	+ 49	+205	+ 51	+190
8·13	3·5	+ 95	+167	+ 92	+154
9·12	2·8	+129	+122	+124	+118
9·79	1·84	+159	+ 88	+157	+ 82
10·0	0·7	+199	+ 42	+190	+ 40

Galvanic Coil.

For attracted point.		Coil with iron core.		Coil without core.	
Longitudinal ordinate.	Transversal ordinate.	Longitudinal force.	Transversal force.	Longitudinal force.	Transversal force.
0·0	2·26	−310	0	− 216	0
1·34	2·26	−286	+169	− 240	+ 38
2·68	2·26	−252	+327	− 315	+ 120
4·02	2·26	−193	+477	− 450	+ 325
5·36	2·26	− 42	+632	− 550	+ 848
6·7	2·26	+162	+562	− 80	+1480
7·78	1·70	+380	+420	+1010	+1630
8·2	0·74	+790	+296	+2480	+1170
0·0	3·73	−164	0	− 134	0
1·34	3·73	−162	+ 82	− 189	+ 41
2·68	3·73	−124	+149	− 200	+ 104
4·02	3·73	− 88	+201	− 217	+ 212
5·36	3·73	− 19	+226	− 123	+ 383
6·7	3·73	+ 39	+214	− 57	+ 424
7·83	3·44	+ 94	+176	+ 100	+ 436
8·82	2·8	+134	+149	+ 264	+ 410
9·49	1·82	+186	+ 99	+ 475	+ 338
9·7	0·73	+223	+ 44	+ 668	+ 186

It does not appear possible to infer from these numbers, by any direct analytical process, the law of distribution of magnetism in the bar. It must be done, I believe, synthetically, by assuming a law, and computing the forces which would result from that law, and then comparing these computed forces with the forces actually observed. The only law which I have tried is the supposition that the intensity of magnetism is proportional to the distance from the centre of the magnet, which includes also the laws that there is a gradual increase of red magnetism from one end and a gradual increase of blue magnetism from the other end. Putting l for the half-length of the magnet, a and b for the longitudinal and transversal ordinates of the attracted point, x for the longitudinal ordinate (measured from the centre) of any attracting point, and supposing the magnet to be a line, it is easily seen that the quantities to be integrated are:—

$$\text{Longitudinal } \frac{-x(x-a)}{\{(x-a)^2+b^2\}^{\frac{3}{2}}}, \quad \text{Transversal } \frac{-bx}{\{(x-a)^2+b^2\}^{\frac{3}{2}}};$$

and the results of integration are:—

$$\text{Longitudinal force } = \frac{l}{\{(l+a)^2+b^2\}^{\frac{1}{2}}} + \frac{l}{\{(l-a)^2+b^2\}^{\frac{1}{2}}}$$

$$+\text{hyp. log}\left\{\overline{(l+a)^2+b^2}\right|^{\frac{1}{2}}-l-a\} - \text{hyp. log}\left\{\overline{(l-a)^2+b^2}\right|^{\frac{1}{2}}+l-a\},$$

$$\text{Transversal force } = \frac{-al-a^2-b^2}{b\{(l+a)^2+b^2\}^{\frac{1}{2}}} + \frac{-al+a^2+b^2}{b\{(l-a)^2+b^2\}^{\frac{1}{2}}}.$$

I have computed these numbers for each of the eighteen stations. For comparison with observation, I have taken the experiments with the flat side towards the small magnets, which represents most nearly the case of a linear large magnet; and, for facility of comparison, I have multiplied the experimental numbers by 6. The following is the comparison:—

Experimental.		Theoretical.	
Longitudinal.	Transversal.	Longitudinal.	Transversal.
−1500	0	−1849	0
−1560	+ 822	−1750	−1089
−1416	+1656	−1441	−2112
−1092	+2664	− 827	−2928
− 216	+3240	+ 155	−3180
+ 948	+2706	+1126	−2283
+1950	+1890	+1589	−1389
+3312	+1104	+2395	− 622
− 960	0	−1029	0
− 894	+ 432	− 971	+ 517
− 738	+ 816	− 776	+ 960
− 474	+1110	− 428	+1267
− 72	+1302	− 2	+1319
+ 306	+1140	+ 335	+1066
+ 552	+ 924	+ 409	+ 801
+ 720	+ 708	+ 668	+ 633
+ 942	+ 492	+ 805	+ 380
+1140	+ 240	+ 984	+ 251

The agreement is not satisfactory; but I am unable to suggest the nature of the change that ought to be made in the assumed law.

I shall add only one remark, of a somewhat practical character. In a paper published originally by Dr. LAMONT in POGGENDORFF'S 'Annalen,' vol. cxiii. p. 239 &c., and of which a translation, by W. T. LYNN, Esq., of the Royal Observatory, is printed in the Philosophical Magazine, 1861, November, Dr. LAMONT inferred the proportion of the effects of different steel magnets from the proportion of the effects of different soft iron bars under the influence of induction. The remark No. 4 (above) goes far, I think, to justify this assumption.

APPENDIX.

Remarking the singularity of the experimental result as to the apparent localization of the attractive pole of a galvanic coil at the very extremity of the coil, I commenced an investigation of the theoretical attraction of a coil, on the laws of galvanic attraction usually received. On my mentioning the subject to my friend JAMES STUART, Esq., Fellow of Trinity College, Cambridge, he kindly undertook, at my request, to prepare a complete theoretical investigation. I am happy in being permitted by Mr. STUART to place before the Royal Society his mathematical discussion of the attraction of the coil, which I am confident will be found to be very complete and of great elegance. I append to it a comparison of the numerical results of the theory with the numerical results of experiment; and the agreement will be found to be so great as to justify entire confidence in the assumed law of galvanic action and the mathematical treatment of it, and a high estimate of the accuracy of the experimental observations.

Investigation of the Attraction of a Galvanic Coil on a small Magnetic Mass.

By JAMES STUART, *M.A., Fellow of Trinity College, Cambridge.*

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From investigations given by AMPÈRE, we can deduce an expression for the potential U at an external point Q of a closed circular galvanic current carried by a wire of indefinitely small section. Let a be the radius of the circle, let the distance of Q from C, the centre of the circle, be r, and let the line C Q make an angle θ with the normal to the plane of the circle: then it can be shown that when r is less than a,

$$U=2\pi k\left\{-1+\frac{r}{a}P_1-\frac{1}{2}\frac{r^3}{a^3}P_3+\frac{1\,.\,3}{2\,.\,4}\cdot\frac{r^5}{a^5}\,.\,P_5-\ldots\right\};$$

and when r is greater than a,

$$U=2\pi k\left\{-\frac{1}{2}\frac{a^2}{r^2}P_1+\frac{1\,.\,3}{2\,.\,4}\cdot\frac{a^4}{r^4}P_3-\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}\cdot\frac{a^6}{r^6}P_5+\ldots\right\},$$

* Abbreviated from the Appendix originally presented and read with the paper.

where k depends only on the intensity of the current, and where P_1, P_3, P_5 are defined by the equation

$$\frac{1}{\sqrt{1-2x\cos\theta+x^2}}=1+P_1x+P_2x^2+P_3x^3+\ldots.$$

If, therefore, X represents the resolved part, perpendicular to the plane of the circle and towards it, of the force exerted by the current on a unit of magnetism placed at Q, and if Y represent the resolved part of that force parallel to the plane of the circle and directed from its centre outwards, then

$$X=\frac{dU}{r\,.\,d\theta}\sin\theta-\frac{dU}{dr}\cos\theta,$$

$$Y=\frac{dU}{r\,.\,d\theta}\cos\theta+\frac{dU}{dr}\sin\theta.$$

To calculate these quantities, we know that

$$P_1=\cos\theta,$$
$$P_2=\tfrac{5}{2}(\cos^3\theta-\tfrac{3}{5}\cos\theta),$$
$$P_3=\tfrac{63}{8}(\cos^5\theta-\tfrac{10}{9}\cos^3\theta+\tfrac{15}{63}\cos\theta).$$

We shall only consider the case of those points for which r is greater than a. Substituting these values in the expression which in such instances holds for U, we have

$$U=2\pi k\left\{-\tfrac{1}{2}\,.\,\frac{a^2}{r^2}\cos\theta+\tfrac{15}{16}\,.\,\frac{a^4}{r^4}(\cos^3\theta-\tfrac{3}{5}\cos\theta)\right.$$
$$-\tfrac{315}{128}\,.\,\frac{a^6}{r^6}(\cos^5\theta-\tfrac{10}{9}\cos^3\theta+\tfrac{15}{63}\cos\theta)$$
$$\left.+\ldots\right\}.$$

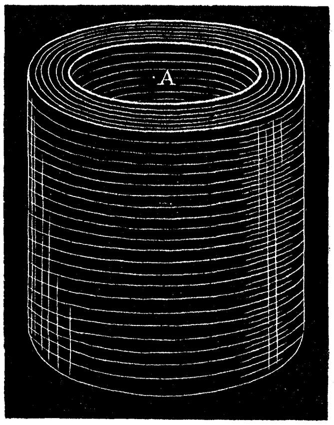
From which, after some reduction, we obtain

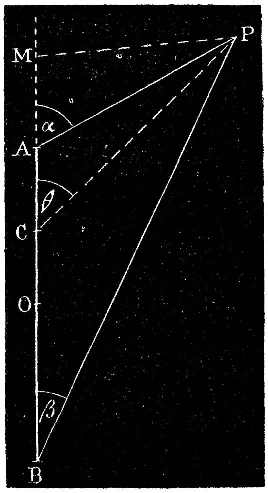
$$\frac{X}{2\pi k}=-\tfrac{1}{2}(-1+3\cos^2\theta)\frac{a^2}{r^3}+\tfrac{1}{16}\,.\,(9-90\cos^2\theta+105\cos^4\theta)\frac{a^4}{r^5}$$
$$-\tfrac{1}{128}(-75+1575\cos^2\theta-4725\cos^4\theta+3465\cos^6\theta)\frac{a^6}{r^7}$$
$$+\ldots\ldots, \qquad (1)$$

$$\frac{Y}{2\pi k}=\sin\theta\,.\left\{+\tfrac{3}{2}\cos\theta\,.\,\frac{a^2}{r^3}-\tfrac{1}{16}(-27\cos\theta+105\cos^3\theta)\frac{a^4}{r^5}\right.$$
$$+\tfrac{1}{128}(525\cos\theta-3150\cos^3\theta+3465\cos^5\theta)\frac{a^6}{r^7}$$
$$\left.-\ldots\ldots\right\}. \qquad (2)$$

Each of these expressions consists of a series of terms in ascending powers of $\frac{a}{r}$, which will be converging.

We shall now seek to find X and Y for a galvanic current traversing a wire coiled into the form of a hollow cylinder, of which the internal radius is b, the external radius $b+c$, and the length is $2f$. We shall suppose the individual turns of the wire to lie so close as that each may be regarded as an exact circle.





Let A B be the axis of the coil, so that A and B are the centres of its two faces; then $AB=2f$. Let O be the middle point of A B. Let P be the attracted point, P M its perpendicular distance p from A B. Let $PAM=\alpha$, $PBM=\beta$.

Let C be the centre of any turn of the wire regarded as a circle of radius a, $CP=r$, $PCM=\theta$, $OC=x$; then it is readily seen that for the whole cylindrical bobbin the forces X, Y are given by

$$\frac{X}{\mu}=\int_{-f}^{+f}\int_{b}^{b+c}\text{L}\,dx\,da,$$

$$\frac{Y}{\mu}=\int_{-f}^{+f}\int_{b}^{b+c}\text{M}\,dx\,da,$$

where L and M stand for the expressions on the right-hand side of (1) and (2) respectively, and where μ depends on the strength of the current.

To perform the integrations for the length of the bobbin in these expressions, we have the formulæ

$$p=r\,.\sin\theta,$$

$$\delta x\,.\sin\theta=-r\,.\,\delta\theta;$$

$$\therefore\ \delta x=\frac{-p\delta\theta}{\sin^2\theta},$$

and

$$r=\frac{p}{\sin\theta}.$$

Making these substitutions for δx and r, the integrals with respect to x become integrals with respect to θ, which can be easily evaluated by a continued application of the method of integration by parts, the limits being from $\theta=\alpha$ to $\theta=\beta$. If we then integrate the result thus obtained with respect to a, from the limit b to the limit $b+c$, we finally obtain

$$\frac{X}{\mu}=\frac{\overline{b+c}^3-b^3}{6p^2}\{-(\cos\beta-\cos\alpha)+(\cos^3\beta-\cos^3\alpha)\}$$

$$+\frac{\overline{b+c}^5-b^5}{80p^4}\{-\quad 9(\cos\beta-\cos\alpha)+33(\cos^3\beta-\cos^3\alpha)$$
$$-\quad 39(\cos^5\beta-\cos^5\alpha)+15(\cos^7\beta-\cos^7\alpha)\}$$

$$+\frac{\overline{b+c}^7-b^7}{896p^6}\{-\quad 75(\cos\beta-\cos\alpha)+\ 575(\cos^3\beta-\cos^3\alpha)$$
$$-1590(\cos^5\beta-\cos^5\alpha)+2070(\cos^7\beta-\cos^7\alpha)$$
$$-1295(\cos^9\beta-\cos^9\alpha)+\ 315(\cos^{11}\beta-\cos^{11}\alpha)\}$$
$$+\ldots\ldots,$$

$$\frac{Y}{\mu}=\frac{\overline{b+c}^3-b^3}{6p^2}\{+(\sin^3\beta-\sin^3\alpha)\}$$

$$+\frac{\overline{b+c}^5-b^5}{80p^4}\{-\ 12(\sin^5\beta-\sin^5\alpha)+15(\sin^7\beta-\sin^7\alpha)\}$$

$$+\frac{\overline{b+c}^7-b^7}{896p^6}\{+120(\sin^7\beta-\sin^7\alpha)-420(\sin^9\beta-\sin^9\alpha)+315\,(\sin^{11}\beta-\sin^{11}\alpha)\}$$
$$+\ldots\ldots$$

These expressions for X and Y will be converging for all points situated at a greater distance than $b+c$ from any point of the axis AB, inasmuch as they are composed by adding together corresponding terms of series which are then all convergent. Among other points, these expressions hold for such as are situated on the axis external to the bobbin, and not nearer A or B than by the distance $(b+c)$. For such points, however, the expressions become illusory, assuming the form $\frac{0}{0}$; they may, however, be evaluated by the methods for the evaluation of vanishing fractions. Y is clearly zero. X may be more readily obtained directly from the expression for U; from that expression we find that for a single circular current the attraction on such points is

$$X=2\pi k\left\{+\frac{a^2}{r^3}-\frac{3}{2}\frac{a^4}{r^5}+\frac{15}{8}\frac{a^6}{r^7}-\ldots\right\}.$$

Hence, in the case of a bobbin, if x be the distance of the attracted point from O, the middle point of the axis of the bobbin, we have

$$\frac{X}{\mu}=\int_{x+f}^{x-f}\int_b^{b+c}drda\left(+\frac{a^2}{r^3}-\frac{3}{2}\frac{a^4}{r^5}+\frac{15}{8}\frac{a^6}{r^7}-\ldots\right)$$
$$=-\frac{\overline{b+c}^3-b^3}{6(x^2-f^2)^2}\left(\overline{x+f}^2-\overline{x-f}^2\right)$$
$$+3\frac{\overline{b+c}^5-b^5}{40(x^2-f^2)^4}\left(\overline{x-f}^4-\overline{x-f}^4\right)$$
$$-5\frac{\overline{b+c}^7-b^7}{112(x^2-f^2)^6}\left(\overline{x+f}^6-\overline{x-f}^6\right)$$
$$+\ldots\ldots,$$

which gives X for points situated on the axis for which x is not less than $(b+c+f)$.

The expressions for forces which concern us now are those given by the general formulæ for $\frac{X}{\mu}$ and $\frac{Y}{\mu}$. And a moment's glance at these will show that they explain the apparent position of the pole at the very extremity of the coil: for, in order to ascertain the values of the forces in a plane at right angles to the axis passing through the extremity of the coil, we must make $\alpha=90°$, $\sin\alpha=1$, $\cos\alpha=0$; and if the other end of the coil be very distant, β may be taken $=0$, $\sin\beta=0$, $\cos\beta=1$. Substituting these values, it will be seen at once that X, the longitudinal force, $=0$, while Y, the transversal force, has a value, which indicates a force directed to the extremity of the coil.

In order to make a complete comparison, I have, for all the eighteen stations treated in the former Tables, taken the values of α, β, and p graphically. For b I have adopted 0·45, and for $b+c$ 0·7; these numbers correspond to the internal and external surfaces of the coil, but they appear to me best to represent (though doubtless with some inaccuracy) the quantities used in the theoretical investigation. Then I have (with the kind assistance of Edwin Dunkin, Esq., of the Royal Observatory) made the complete calculation of the formulæ for every station. As the numbers first obtained were not immediately comparable, I have made them more nearly so by trebling the numbers given by theory and doubling those in the preceding Table. The results are as follows:—

Longitudinal ordinate.	Transversal ordinate, or p.	α.	β.	Result of theoretical calculation.		Theoretical result trebled.		Experimental result doubled.	
				X.	Y.	X.	Y.	X.	Y.
0·0	2·26	161° 10′	18° 40′	− 160	0	− 480	0	− 432	0
1·34	2·26	157 15	15 40	− 168	+ 30	− 504	+ 90	− 480	+ 76
2·68	2·26	151 0	13 25	− 208	+ 82	− 624	+ 246	− 630	+ 240
4·02	2·26	140 20	11 35	− 297	+206	− 891	+ 618	− 900	+ 650
5·36	2·26	121 55	10 15	− 354	+503	−1062	+1509	−1100	+1696
6·7	2·26	91 0	9 20	− 38	+855	− 114	+2565	− 160	+2960
7·78	1·70	58 25	6 10	+ 543	+771	+1629	+2313	+2020	+3260
8·2	0·74	28 0	2 35	+1417	+688	+4251	+2064	+4960	+2340
0·0	3·73	151 0	29 0	− 124	0	− 372	0	− 368	0
1·34	3·73	145 30	24 42	− 128	+ 32	− 384	+ 96	− 378	+ 82
2·68	3·73	137 20	21 30	− 139	+ 77	− 417	+ 231	− 400	+ 208
4·02	3·73	126 10	18 57	− 150	+148	− 450	+ 444	− 434	+ 424
5·36	3·73	110 5	16 58	− 109	+243	− 327	+ 729	− 246	+ 766
6·7	3·73	91 0	15 27	− 20	+295	− 60	+ 885	− 114	+ 848
7·83	3·44	73 0	13 2	+ 80	+308	+ 240	+ 924	+ 200	+ 872
8·82	2·8	54 5	10 0	+ 179	+281	+ 537	+ 843	+ 528	+ 820
9·49	1·82	34 35	6 10	+ 318	+223	+ 954	+ 669	+ 950	+ 676
9·7	0·73	14 25	2 5	+ 456	+120	+1368	+ 360	+1336	+ 372

In spite of some discordances in the large forces (which it was impossible to measure with accuracy), there is enough of agreement to show that confidence may be placed in the method of theoretically computing the attraction of the galvanic coil.